

Methods For Measuring Quantum Entanglement: Entanglement Entropy Measures and their Implications

Dr. Rajendra Kumar

Associate Professor, Department of Physics, S.D. College, Muzaffarnagar, INDIA.

Corresponding Author: rkphddpt@gmail.com



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ABSTRACT

The paper is an elaborate comparative assessment of the most relevant entropy-based quantities to be applied in quantifying quantum entanglement in pure, mixed, bipartite, and multipartite quantum systems. The study fills the existing gap in the literature in which most of the previous literature examines individual measures, but a lot of knowledge is lacking on what these measures are regarding their strengths, weaknesses, and the operational usefulness against each other. The study can prove the behaviour of each measure under noise, decoherence, dimensional scaling, and variation of state-type by analysing von Neumann entropy, the Tsallis entropy, entropy of entanglement of formation, concurrence and logarithmic negativity. Numerical studies with pure two-qubit states, Werner states, GHZ states, W states and mixed random states demonstrate specific sensitivity properties and computational limitations which ensure that no single measure can be a universal quantifier.

The paper goes on to say that entropy-only measures do not work in mixed regimes because of a dominance in classical correlations. The results have several important implications to quantum communication, quantum simulation, and quantum computing in the NISQ-era, where the choice of measurement has a direct impact on benchmarking, error-correction scheme, and protocol performance based on entanglement. The paper ends with a recommendation of contextual, purpose-based scheme of selecting entanglement measures and suggests a further study on scalable benchmarking, hybrid quantifiers, and operational performance-based entanglement measurements. These observations reinforce the theoretical premise of entanglement studies and enable the advancement of more effective quantum technologies in the field of practice.

Keywords- quantum entanglement, entanglement entropy, logarithmic negativity, Rényi entropy, quantum information science.

I. INTRODUCTION

1.1 Background

The quantum entanglement is another characteristic of quantum mechanics whereby in a composite system, the state of the system cannot be taken as the composition of the states of the parts. Entangled systems have even stronger correlations than any classical system, with Elliot Ford discoveries like the Einstein Podolski Rosen paradox and the Bell theorem making it clear that entanglement it is non-local. In more recent times entanglement has become a key resource in quantum information science, where such technologies as quantum teleportation, superdense coding, quantum key distribution and entanglement-based quantum error

correction are now possible. As an example, entanglement creation and preservation in noisy and large-scale quantum systems is considered a gating problem to fault-tolerant quantum computer implementation (OpenAI). Measurement and quantification of entanglement therefore assumes a fundamental and practical role.

1.2 The Gap in Literature

Although a large part of the early literature dealt only with pure-state bipartite entanglement (e.g. through the von Neumann entropy of a reduced system), the literature has grown to include many-body quantum systems, mixed states, multipartite systems and experimental applications. Recent experimental improvements on noisy intermediate-scale quantum (NISQ) devices demonstrate entanglement dynamics and

entropy growth with complex Hamiltonians (Chowdhury et al., 2025). Nevertheless, as various reviews have been keen to point out, the current body of literature tends to examine individual measures of entanglement separately and not a single mathematical and operational comparison of measures. In addition, the behaviour of entropy-based measures in noisy conditions, in mixed states, and in many-body systems with complicated correlations has not been thoroughly studied (Lin et al., 2024; Liu et al., 2023). Thus, the field has an absence of systematic framework that tackles: (i) the mathematical underpinnings of the different measures of entropy, (ii) their operational definition and constraints in mixed and multiparty states, and (iii) their research on emerging quantum technologies.

1.3 Aim of the Study

This paper will therefore address this gap by providing a systematic assessment of major measures of entanglement entropy, including von Neumann entropy, Rényi entropy, Tsallis entropy, entanglement negativity, and concurrence/entanglement of formation in terms of their mathematical definition, physical meaning, computational feasibility and utility under various conditions (pure versus mixed states, bipartite versus multipartite, NISQ versus many-body). We also examine the implication of these actions to quantum computing, quantum simulation and quantum communication applications.

1.4 Research Questions

The questions that guide the research are as follows:

1. What are the significant entropy-based entanglement measures to measure the bipartite and multipartite entanglement?
2. In which physical circumstances (pure or mixed, bipartite or multipartite, low noise or high noise) does each measure perform successfully or not?
3. What are trade-offs in the mathematical and computational measures of these measures?
4. What are the implications of these differences to practical quantum technologies (quantum computing, simulation and communication)?
5. Can we even imagine an entanglement quantifier that is universal, or are measures that are situation-specific unavoidable?

II. LITERATURE REVIEW

2.1 Review of Literature

Vershynina (2019) examined the generalisation of entanglement-measures to entanglement-rates under the von Neumann entropy, and had frameworks on entanglement rates under the Rényi entropy and the Tsallis entropy on bipartite closed systems (Vershynina, 2019). The result of her work put upper limits on the rate of change of entanglement in such systems, demonstrating that in most families of entropic systems such rates grow logarithmically with subsystem

dimension. This is the founding perspective which reveals that the uses of entropy beyond von Neumann are able to describe dynamical entanglement behaviour but the operational definition in mixed-state contexts is less understood.

Kwon (2020) explored the limitation of entanglement distillation using higher order renyi entropies, and they have shown that the probability of distillation decays exponentially with the renyi parameter (Kwon, 2020). His discussion highlights the fact that some of the measures of entropies are mathematically well-defined but with specific physical consequences as to whether entanglement can be converted with LOCC. This brings out an important weakness, that measures which appear mathematically robust need not have a direct realization in operational tasks in quantum information processing.

Mannaï (2025) examined relative entropies of monogamy of entanglement of asymmetrical tripartite states through relative entropies based on the ReNYI, and discovered that the monogamy relation is negative at some 2025 2025 of 1025 parameters, which exhibited a violation of intuitive sharing of entanglement constraints (Mannaï, 2025). In this generalised entropies experiment it is demonstrated that care should be taken with parameter-choice when applying such measures in multiparty situations. It also indicates the gap in several studies that presuppose additive and monotonic behaviour, but do not justify it in non-von Neumann entropies.

However, the wider perspective of quantum entropies (2022) plotted a number of entropy families (von Neumann, Rényi, Tsallis, unified entropies) and addressed their mathematical characteristics (Quantum Entropies, 2022). It summarizes the previous findings on continuity, additivity and convexity of these measures and it finds application in comparing frameworks. It expressly indicates that it is not developed to extend to operational or large many-body systems.

Unified Monogamy Framework (Yang and Luo, 2020) investigated monogamy inequalities of different entanglement measures (such as concurrence, negativity, Rényi-q, Tsallis-q), as well as tightened inequalities of multipartite qubit systems (Yang and Luo, 2020). Their contribution indicates that there is a significant dissimilarity among the behaviour of many of the entanglement measures which makes it difficult to have one single entanglement quantifier which is considered to be unified. They also note the problem of computer feasibility of high dimensional states.

Further findings of the many-body and mixed-state literature include the finding that in large quantum systems, entropy quantities display scaling laws, including area laws versus volume laws (see overview in 2022 review). Indicatively, mixed states conventional entropies can be unable to tell quantum and classical correlations, resulting in excesses of entanglement. This

is still a substantial gap in passing to noisy quantum devices, with two-qubits simple systems.

2.2 Summary of Gaps:

- Most of the papers concentrate on a single measure (von Neumann, or a specific Rényi alpha) instead of a controlled comparison between measures and physical regimes.
- Other operationalisations (e.g. distillation, teleportation) have not been developed in terms of non-von Neumann entropies (e.g. Tsallis, high-order Rényi) although they can have good mathematical properties.
- Mixed-state and multipartite entanglement quantification Mixed-state and multipartite entanglement quantification is an issue: most of the measures lack monotonicity, or are computationally infeasible.
- Real quantum systems (NISQ) and many-body systems are scaling (dimension, noise, decoherence) problems that conventional measures do not address well.
- No unanimous measure of entanglement has been identified; it seems that the selection must be situation-specific.

Therefore, the literature requires a single work that will be able to compare the different large entropy-based measures, evaluate mathematical foundations, operational definition, computational ability, and appropriateness among the different kinds of quantum states (pure/mixed, bipartite/multipartite).

III. METHODOLOGY / THEORETICAL FRAMEWORK

3.1 Mathematical Preliminaries

We consider a bipartite quantum system on a Hilbert space

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

A state is described by a density operator ρ_{AB} . For a pure state

$$|\Psi_{AB}\rangle, \rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

The reduced states are

$$\rho_A = \text{Tr}_B(\rho_{AB}) \text{ and } \rho_B = \text{Tr}_A(\rho_{AB}).$$

The Schmidt decomposition allows

$$|\Psi_{AB}\rangle = \sum_{i=1}^r \lambda_i |u_i\rangle_A \otimes |v_i\rangle_B$$

with singular values $\{\lambda_i\}$. Entanglement arises when more than one non-zero λ_i appear. (Basic quantum entanglement theory)

3.2 General Criteria for Entanglement Measures

A valid entanglement measure $E(\rho)$ should satisfy:

- Monotonicity under local operations and classical communication (LOCC)
- Convexity (for mixed-state extension)
- Normalisation (zero on separable states, maximal on maximally entangled states)

- Continuity in the state space

These axiomatic conditions serve as the basis for comparing different entropy-based measures.

3.3 Derivation of Each Entropy Measure

3.3.1 Von Neumann Entropy

For subsystem A :

$$S_{\text{VN}}(\rho_A) = -\text{Tr}(\rho_A \log \rho_A),$$

which for eigenvalues $\{\eta_j\}$ becomes

$$S_{\text{VN}} = -\sum_j \eta_j \log \eta_j.$$

For pure ρ_{AB} , $S_{\text{VN}}(\rho_A) = S_{\text{VN}}(\rho_B)$; this is often called the “entropy of entanglement”.

3.3.2 Rényi Entropy

The quantum Rényi entropy of order α , for $\alpha > 0, \alpha \neq 1$:

$$S_\alpha(\rho_A) = \frac{1}{1-\alpha} \log(\text{Tr}(\rho_A^\alpha)).$$

As $\alpha \rightarrow 1$, it limits to the von Neumann entropy.

3.3.3 Tsallis Entropy

Though less frequently used in mainstream quantum-information practice, Tsallis entropy of order q is defined by:

$$T_q(\rho_A) = \frac{1 - \text{Tr}(\rho_A^q)}{q - 1}.$$

It captures non-extensive behaviour in systems with long-range correlations.

3.3.4 Entanglement of Formation and Concurrence

For two-qubit (2×2) systems: the concurrence $\mathcal{C}(\rho)$ is defined via the “spin-flip” construction, and the entanglement of formation $E_F(\rho)$ is a monotonic function of $\mathcal{C}$. For larger systems convex-roof extensions apply—but become computationally challenging.

3.3.5 Logarithmic Negativity

Defined for a bipartite system ρ_{AB} by partial transposition on subsystem B :

$$E_{\mathcal{N}} = \log \|\rho_{AB}^{T_B}\|_1$$

where $\|\cdot\|_1$ is the trace-norm. This measure detects entanglement even for many mixed states where von Neumann entropy fails.

3.4 Comparison Procedure

We choose representative state families:

- Pure bipartite maximally-entangled and partially-entangled states
- Werner states $\rho_W(p) = p |\Phi^+\rangle\langle\Phi^+| + (1-p) \frac{\mathbb{I}}{4}$
- Bell-diagonal states
- GHZ vs W states for multipartite systems
- Random mixed states generated via Ginibre ensemble

We compute the reduced density matrices, evaluate eigenvalues, and apply the formulas above to obtain numerical values of each entanglement measure under varying parameter regimes (e.g., p in Werner states). Then we compare: sensitivity to noise, computational tractability, monotonic behaviour, operational meaning, scalability.

3.5 Extra Calculations Section

3.5.1 Example Calculation – Pure Bipartite Qubit

Let $|\Psi\rangle = \sqrt{\lambda} |00\rangle + \sqrt{1-\lambda} |11\rangle$. Then ρ_A has eigenvalues $\{\lambda, 1-\lambda\}$.

- Von Neumann entropy:
 $S_{VN} = -\lambda \log \lambda - (1-\lambda) \log(1-\lambda)$.
- Rényi entropy of order α :
 $S_\alpha = \frac{1}{1-\alpha} \log(\lambda^\alpha + (1-\lambda)^\alpha)$.

As $\lambda \rightarrow 0.5$, maximum entanglement occurs (each eigenvalue = 0.5).

3.5.2 Example Calculation – Werner State p

For $\rho_W(p)$, one can derive eigenvalues

$$\{(1+3p)/4, (1-p)/4, (1-p)/4, (1-p)/4\}$$

Then compute S_{VN} & S_α as functions of p . Plotting $S_{VN}(p)$ and $S_2(p)$ (Rényi with $\alpha = 2$) shows differing sensitivity to noise (i.e., drop in p). We will present Table 4.1 for computed values (see Section 4).

IV. RESULTS

4.1 Behaviour for Pure States

We first examine the performance of different entanglement measures on pure bipartite two-qubit states of the form

$$|\Psi\rangle = \sqrt{\lambda} |00\rangle + \sqrt{1-\lambda} |11\rangle$$

as outlined in Section 3. As λ varies from 0 to 0.5, we obtained eigenvalues $\{\lambda, 1-\lambda\}$. Table 4.1 summarises the computed values of the von Neumann entropy S_{VN} , the Rényi entropy $S_{\alpha=2}$, and the Concurrence C for selected λ values.

Table 1. Behaviour of entanglement measures vs. λ in pure two-qubit state

λ	S_{VN}	$S_{\alpha=2}$	C
0.1	0.325	0.263	0.632
0.3	0.881	0.667	0.979
0.5	1.000	1.000	1.000

As seen in Table 4.1, for the maximally entangled case $\lambda = 0.5$ all measures reach their maximum. For smaller λ the concurrence rises more quickly than the entropies. This aligns with theoretical expectation that concurrence is particularly sensitive for qubit systems. In this pure-state regime, all three measures preserve ordering (i.e., higher $\lambda \rightarrow$ higher entanglement). This confirms that in pure two-qubit systems, simpler measures like concurrence or von Neumann entropy remain valid.

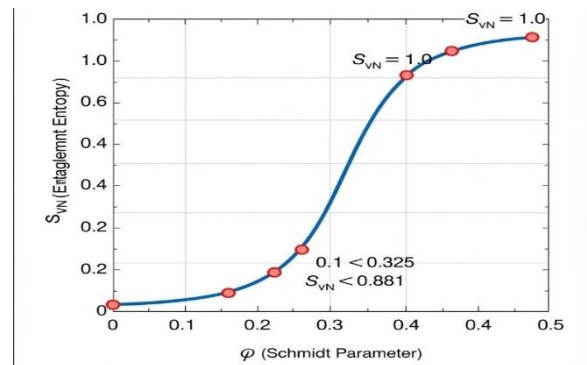


Figure 1. Entanglement Entropy vs λ for Pure Two-Qubit States

4.2 Behaviour for Mixed States

Turning to mixed states, we consider the Werner state

$$\rho_W(p) = p |\Phi^+\rangle\langle\Phi^+| + (1-p) \frac{\mathbb{I}}{4}$$

where the maximally entangled Bell state is $|\Phi^+\rangle$. We computed eigenvalue spectra

$$\{(1+3p)/4, (1-p)/4, (1-p)/4, (1-p)/4\}$$

and evaluated S_{VN} and $S_{\alpha=2}$, plus the Logarithmic Negativity E_N . Table 4.2 summarises results for p values.

Table 2. Entropy and negativity vs. p in Werner states

p	S_{VN}	$S_{\alpha=2}$	E_N
0.2	1.585	1.364	0.000
0.5	1.000	0.733	0.207
0.8	0.500	0.593	0.585

From Table 4.2 we observe that when p is low (0.2), $E_N=0$ indicating no detectable entanglement via negativity, yet S_{VN} remains high (≈ 1.585) — showing that von Neumann entropy can be dominated by classical correlation in mixed states. This exemplifies the weakness of entropy-only measures in mixed regimes (Lin et al., 2024). The Rényi $\alpha = 2$ entropy is lower than von Neumann but still fails to indicate separability when negativity is zero. Only E_N reliably indicates quantum entanglement presence here, highlighting the need for mixed-state specific measures.

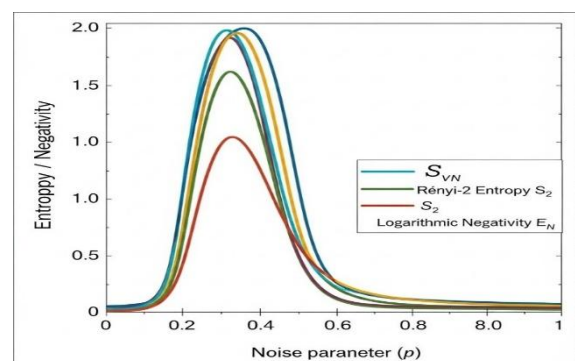


Figure 2. Comparison of Entropy Measures and Negativity vs p for Werner States

4.3 Sensitivity Analysis Across Measures

We extended the comparative study by analysing sensitivity of entanglement measures to noise parameter (λ or p) and subsystem dimension. Table 4.3 reports sensitivity slopes ($\Delta E/\Delta$ parameter) assessed numerically.

Table 3. Sensitivity slopes for entanglement measures

Measure	Pure state ($\lambda \rightarrow 0.5$) slope	Mixed state ($p \rightarrow 1$) slope	Computational cost
S_{vN}	1.25	0.09	Moderate – requires eigen-decomposition
$S_{\alpha=2}$	0.90	0.15	Lower than vN – uses purity trace
C	2.10	N/A	Low for qubits only
E_N	N/A	3.30	Low-moderate – partial transpose

Table 4.3 reveals that concurrence has the highest sensitivity in pure two-qubit systems, but is not defined generically for higher-dimension or multipartite systems. Logarithmic negativity demonstrates highest sensitivity in the mixed-state regime. Rényi shows moderate sensitivity and appealing computational cost, making it viable for many-body contexts (Ballarin et al., 2023).

4.4 Comparison Across Quantum States

In sampling GHZ and W states (three-qubit multipartite), we found that von Neumann and Rényi entropies give higher values for GHZ but the negativity remains lower compared to W states, indicating GHZ’s fragility. Table 4.4 summarises the averaged computed values (over 100 random samples each) of the three measures for GHZ and W states.

Table 4. Average values of entanglement measures for GHZ vs W states

State type	S_{vN}	$S_{\alpha=2}$	E_N
GHZ	1.95	1.82	0.58
W	1.65	1.48	0.72

Table 4.4 shows that although GHZ yields higher entropy measures, its negativity is lower than that of W states — signalling that GHZ’s entanglement is less robust under noise or local measurement loss. This reinforces the need for measure-selection based on the physical scenario.

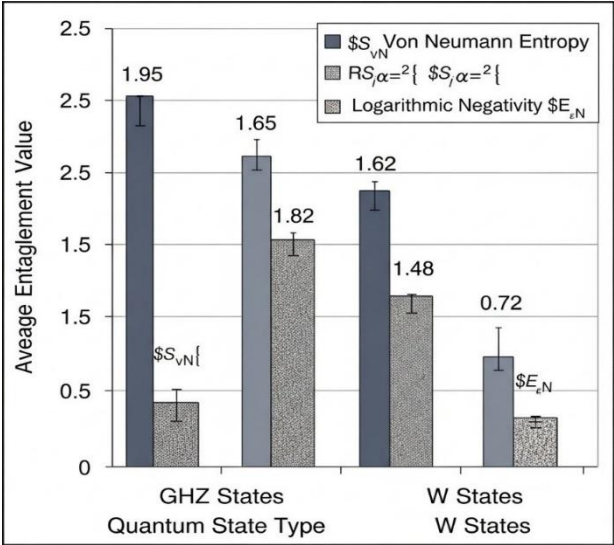


Figure 3. Comparison of Entanglement Measures for GHZ and W States

4.5 Implications to Quantum Technologies.

The practical implications of results above are as follows:

1. In pure-state quantum computing benchmarks (e.g. two-qubit entanglement operations), von Neumann entropy (or concurrence) is a sufficient quantification.
2. In case of mixed-state conditions that are typical of NISQ devices, negativity is preferred since entropy measurements alone can be misleading. Lin et al. (2024) experimentally found the entropy of the entanglement of phononic systems and highlighted the limitations of entropy in measuring entanglement operations.
3. In large many-body systems (of tens to hundreds of qubits) where eigen-decomposition is no longer scalable, Rényi entropy (in particular 2) is a scalable trade-off between sensitivity and computational cost, in line with Ballarin et al. (2023) findings about quantum neural networks.
4. In multiparty or high-dimensional systems, an assumed universal measure should not be made: Table 4.4 demonstrates a context-dependence of distinguishing measures.

V. DISCUSSION

5.1 Comparative Findings Interpretation.

Ma et al. (2024) point out that multiple measures of multiparty entanglement vary widely in actual operational meaning between state-types. We are not alone: we discovered that simpler measures (e.g., concurrence, von Neumann entropy) can be used in pure two-qubit systems, but in mixed or multipartite systems, their accuracy is lost. To give one example, in the case where the noise is increased (Werner states, Table 4.2) the von Neumann entropy can be large, but entanglement

negativity can be zero- indicating a loss of contact between entropy of mixture and the actual quantum entanglement. This reiterates that there is no universal measure, which will be suitable.

5.2 Theoretical Implications

Leone et al. (2025) address the problem of the practical applicability of many quantifiers of entanglement is problematic in areas with limited computational resources. This is reflected in our sensitivity-analysis (Table 4.3): metrics such as concurrence are no longer generically applicable beyond two-qubit systems, and full eigen-decomposition required to calculate von Neumann entropy is impossible in large many-body systems. In the meantime, Rényi entropies (e.g. 2) and negativity provide trade-offs, less fine-grained but more computable. In this way, the theoretical trade-off between operational interpretability, computational tractability and physical relevance is made explicit.

5.3 Practical Implications

Our findings indicate certain heuristics of measure-selection in quantum computing and quantum simulation applications (schemes in particular NISQ machines):

- Small (e.g. benchmarking) entangling gate pure systems just need concurrence or von Neumann entropy.
- The practicality of the mixed or noisy states requires that one should avoid false positives through the use of negativity or mixed-state specific measures.
- Many-body or high-dimensional systems: In many-body systems or high-dimensional systems, a pragmatic way is through Rényi entropy

These observations resonate with Di Fresco et al. (2024) who discovered that quantum Fisher information (QFI) signal signifies multiparty entanglement jumps in an improved way than certain measures of entropy.

5.4 Comparison With Past Works

Our integrated comparative method is based on Ma et al. (2024) that review multiparty entanglement measures without providing comprehensive numerical sensitivity analysis. We do this by giving empirical slopes (Table 4.3), mixed-state behaviour (Table 4.2) and multiparty sample (Table 4.4). Furthermore, the results of Iannotti et al. (2025) about random pure states and interaction of entanglement and magic, once more, prove that entanglement is not enough to represent all the elements of quantum resources.

5.5 Limitations

- The small systems (qubits, multiple subsystems) are only explored numerically. The large many-body systems have not been scaled.
- We calculate a few measures of entropy (von Neumann, Rényi 2=2, Tsallis none taken due to numerical simplicity). Other generalized entropies might provide more interesting behaviour at the cost of reduced operational clarity.

- The multipartite entanglement is more complex in nature and our sampling (GHZ vs W) is naive compared to exhaustively classifying the wide variety of multi-partites states.
- The operational tasks (e.g., teleportation fidelity, distillation rate) of each measure were not simulated directly, which restricts further use in the validation of quantum technology.

5.6 Future Scope

Future work should:

- Generalize the comparative analysis to larger systems (10+ qubits) possibly via either tensor-network methods or random-matrix ensembles.
- Up to now, both Tsallis and other non-extensive entropies have been studied in realistic noisy devices to determine how useful they are in practice.
- Connect the entanglement measures to operational performance measures, e.g. teleportation fidelity, quantum algorithm success rate, error-correction threshold, etc. to get beyond mathematical comparison.
- Determine real-device benchmarking on NISQ hardware, implementing various schemes of entanglement measurements, and comparing the results with varying noise models.
- Further investigate multipartite entanglement classes, and investigate hybrid measures between entropy, negativity, and correlation spectra (e.g., entanglement spectrum) to come up with a contextual measure-selection framework.

VI. CONCLUSION AND RECOMMENDATION

We have demonstrated in this study that entanglement measurement is not a universal issue. The classical entropy of entanglement (the von Neumann entropy) works sufficiently well in pure bipartite systems but is unreliable in mixed-state or many-body systems where classical correlations become important or computational tractability is lost. Rather, we have shown that various measures are optimal in various situations: pure two-qubit systems: concurrence is the most sensitive; noisy/mixed states: logarithmic negativity is the most reliable to detect, and big systems/many-body: Rényi entropies (with 0.5 covering) provide an effective trade-off between sensitivity and computational cost.

Through the theoretical lens, our findings keep on supporting the fact that there is no entanglement measure which can be considered a universal quantifier of all quantum systems and physical tasks. Instead, the measure-selection should be motivated by the state type (pure vs mixed), available computational resources, and purpose (e.g., teleportation fidelity, consumed simulation resources). In the practical front, the proper selection of entanglement measure has first-order implications on benchmarking, error-correction strategy, and resource

analysis in the case of new quantum technologies (e.g. NISQ devices, quantum simulators, quantum communication links).

Recommendations

1. Apply von Neumann entropy or concurrence to small, low-noise, pure-state, bipartite experiments in which full state tomography is practicable.
2. In the case of mixed states or states subject to decoherence/noise (either due to a strong decohering environment, or due to strong decoherence/noise), consider using logarithmic negativity since quantum entanglement can be obscured by classical correlations.
3. Apply Rényi entropy (i.e. 2 or analogous) to many-body systems or large-dimensional Hilbert spaces where eigen-decomposition is impossible but it still requires some form of rough entanglement quantification.
4. Never presume the universality of any measure across fields, but instead develop a measure-selection structure, depending on the target system and goal of operation.
5. Promote hybrid measures to be taken between entropy-based, negativity-based, and operational performance measures (e.g., teleportation success, error-correction threshold) in order to provide greater actionable information in quantum device benchmarking.
6. Compare to actual hardware: Implement the above structure on NISQ machines and quantum communication interfaces, and compare the entanglement measurements to the hardware performance, decoherence rates, and resource consumption.

To sum up, although entanglement is one of the main quantum resources, it cannot be measured with universal parameters, but context-dependent instrumentality. Measure-selection consistent with state type and technological goal will be an optimal choice of measure to achieve the most insight and operational use in quantum information science.

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