

Nonlinear Dynamics and Chaos in Mechanical Oscillatory Systems

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ABSTRACT

Nonlinear dynamics and chaos could not be for complex behaviour. Such behaviour leads to oscillations in mechanical systems. Due to the overwhelming influence of chaotic behaviour in such systems, the present paper was carried out mostly theoretically and computationally-chaos mostly from huffing oscillator and its mechanical implementations. In addition, with the setup of experimental wide-parameters robust chaotic oscillations (Karimov et al., 2023) a new mechanical scheme realization in magnetic springs has been opened. Furthermore, from theoretical point of view there are recent papers that have revealed the transition from periodic to chaotic dynamics via period-doubling and bifurcation in hyperbolic double-well Duffing-type oscillators (Author et al., 2025). Inferring hidden dynamics framework is a new approach to predict observable and hidden variables from complex oscillatory systems (Stepanian's et al., 2024). Therefore, in the paper we have decided to integrate these ideas and recent results from 2023 to 2025, and to carry out the following recent analysis of nonlinear behaviour, have looked at what approaches were used to detect chaos (through bifurcation diagrams, Lyapunov exponents, Poincare maps), and what the implications are for vibration control, fault diagnosis, and energy harvesting.

Keywords- Nonlinear dynamics, Chaos, Duffing oscillator, Bifurcation, Vibration control.

I. INTRODUCTION

There is no modern physics or engineering position that nonlinear dynamics can claim, particularly in oscillatory motion where linear models fail to capture the desired behaviour. In this regard, mechanical oscillatory systems such as the Duffing oscillator have been widely investigated to show that in some regimes of parameters, chaotic behaviour may appear (Karimov et al., 2023).

Actual reasons behind our curiosity for such systems can be traced to the opportunity of using them for control, fault diagnosis, energy harvesting, and optimal mechanical design. The nonlinear subspace identification method was effectively used to characterize a double-well Duffing oscillator with position-dependent friction (Zhu et al., 2022).

Duffing oscillator is one of the simplest nonlinear mechanical systems treated in the literature-weakly nonlinear second order non-autonomous differential equation with nonlinear restoring forces.

Conventional variants of the Duffing oscillator include mechanical and magnetic springs-the latter is now under stress for being robust and wear-free. Karimov et al. (2023) have studied a magnetic-spring Duffing mechanical oscillator admitting chaotic oscillations in wide parameter ranges, thus giving a feasible low-wear implementation for the same. From this setup, new avenues have been opened for the experimental investigation of chaos theory in mechanical systems.

The theoretical improvements of the Duffing model continue to hold sway. Since 2025, Dikandé has introduced the hyperbolic double-well Duffing-type oscillator whose hyperbolic potential adjusts its shape via a deformability parameter, yet the barrier height remains fixed. The system undergoes periodic and chaotic dynamics through period-doubling or period-halving bifurcations as ascertained via Poincaré mapping and bifurcation analysis. In recent times, people have injected mechanical-chaotic paradigms to the area of spintronics. On magnetic Duffing oscillators based on ferromagnets with uniaxial anisotropy: Tatsumi et al. (2025) discovered chaos in magnetization dynamics under periodic external

fields with positive Lyapunov exponents and existence of homoclinic orbits in the phase space! Hence, these works connected nonlinear dynamics with magnetization control and thereby extended the scope of chaos beyond pure mechanical systems.

With regard to the above-mentioned new work, this paper is motivated by two points of high importance:

1. Investigating practical implementation vis-à-vis theoretical modelling: Are Duffing-type oscillators based on magnetic springs having hyperbolic double-well potentials really physically plausible? If so, then the answer is sure to come from further analytical investigation.
2. Looking for relevance for a multidomain spectrum: Chaotic oscillators live in a big area stretching from vibration control in engineering to psionic devices and hence ought to be treated coherently from that descriptive expanse.

The study by Zhu, Marchesiello, Anastasio, Jiang, and Fei (2022) presents an innovative approach for estimating both stiffness and damping parameters in a Duffing-type nonlinear system.

The objectives of the study, consequently, are (a) to investigate and compare the studied dynamics of magnetic-spring-implemented mechanical Duffing oscillators, (b) to study bifurcation phenomena of hyperbolic double-well models with respect to deformability, and (c) to draw wider implications for the proper chaos modelling to be used in control applications and future technologies, such as magnetic oscillatory systems.

Hence, for the achievement of the objectives proposed, the development will thus be conducted following these lines: Section 2 contains a literature review on nonlinear dynamics foundations, mechanical Duffing systems, and modelling; Section 3 shows the methodology: modelling, simulation techniques, and chaos detection tools; Section 4 shows results and analysis related to transitions to chaos, bifurcation diagrams, and comparative studies on system behaviour; Section 5 presents a discussion, interlinking the findings, limitations, and future work proposals; to conclude, Section 6 concludes the paper with reflections on engineering and theoretical aspects. This design is in the right order-from theory and motivation to methods and results to the broader context-aligned with your arrangement above.

II. LITERATURE REVIEW

Due to their fundamental and applied attributes, the behaviour of Duffing oscillators will always remain interesting to scientists, and since the 1950s, works have started appearing at an increasing rate addressing aspects of their theory and physical fundamentals. In this paper, we present a literature review spanning three domains that are closely coupled domains:

- (1) mechanical realisations of the Duffing oscillator, the most notable being magnetic springs;
- (2) extensions of the classical Duffing paradigm, most prominently hyperbolic double well potentials and extensions thereof; and
- (3) analogues across disciplines such as the magnetic Duffing oscillator in spintronics.

Abohamer et al. (2024) provide a comprehensive analysis of the chaotic behavior and stability characteristics of a Duffing oscillator under sinusoidal forcing.

2.1 The Mechanical Duffing Oscillators with Magnetic Springs

Nonlinear spring has been the archetypal and most traditional element in the experimental realisation of mechanical Duffing oscillators. The magnetic springs are recent development magnetic repulsion or magnetic attraction provides a natural nonlinearity with no shear wear or friction (Karimov et al., 2023). Karimov et al. realised a strong mechanical Duffing oscillator with magnetic springs which exhibited chaotic behaviour in simulations and experiments over a broad range of parameters. Some of the advantages of this arrangement include no wear which means no noise and simple d maraging steel section changes.

2.2 Some Theoretical Extensions: Hyperbolic Double-Well and the Generalized Duffing Models

An ordinary Duffing assumes the classical double-well polynomial potential. However, the potential of varied form could provide different dynamical behaviors. Dikandé (2025) introduced a Hyperbolic Double-Well Duffing-type oscillator in which the potential is given by a hyperbolic function with two degenerate minima. The controllable deformability parameter is significant since it can be controlled without the intervention of barrier height. Dikandé showed that when this parameter changes, the oscillator moves from some periodic and chaotic states through period-doubling or period-halving bifurcation. Additionally, studies suggest famous chaos precursors as appeared in the standard Duffing model, from both analytical and numerical viewpoints, especially from Poincaré maps.

Meanwhile, Rondón and Sadri (2025) furthered the foundations of the generalized Duffing oscillators with a complete classification of phase portraits. The study focuses on the centre-focus problem for the generalized Duffing systems given by the differential equations of the form $\dot{x} = y$, $\dot{y} = -\alpha y - \epsilon x^m - \sigma x$, where $m \geq 1$. They found conditions of existence of homoclinic and heteroclinic orbits and ruled out limit cycles under some conditions. These considerations of pure mathematics extend the theoretical background of nonlinear oscillators and enrich the analytical fluid for the identification of chaos. Through analytical and numerical treatment, Abohamer, Amer, Arab, and Galal (2024) illuminate the dynamic regimes of a parametrically forced Duffing oscillator.

2.3 Magnetic Duffing Oscillator: Chaos in Spintronic

In addition to the mechanical analogies, the Duffing framework has recently been transferred to spintronic systems. Tatsumi et al. (2025) proposed the magnetic Duffing oscillator to represent a ferromagnet having a uniaxial anisotropy under the influence of a periodically oscillating external field. In this term 1825 coupled with the Landau Lifshitz Gilbert equation, the double-well magnetic potential was considered to be caused by the anharmonic external field being imposed perpendicular to the anisotropy axis responsible for homoclinic orbits and chaotic magnetization dynamics. Chaos methods of computing include Lyapunov exponents, Poincaré sections, and bifurcation diagrams. And interestingly enough, even the type and parameter range of chaos could be made to depend on the strength of the external magnetic field. As such, this research aims to conceptualize a bridge between chaos theory and spin dynamics, suggesting the presence of Duffing-like behaviour in some current spintronic devices. The use of the LLG equation, being a micromagnetic standard describing magnetization precessions and damping, sustains the physical validity and relevance of the framework.

2.4 Literature Themes

Mechanical Realization Benefits

Karimov et al. (2023) maintain how magnetically generated springs have the qualitative aspect of a real-world and simply designed chaotic-dynamics generator of mechanical oscillators. Hence, this design allows for experimental scenarios that benefit the increased ease of education and applied research. Yang, Ding, Gao, and Chen (2023) utilize a chaotic effect-based array of Duffing systems with enhanced restoring forces to detect weak signals under challenging conditions.

Novel Theoretical Insight

An exceedingly interesting hyperbolic potential model (Dikandé, 2025) and a general classification (Rondón & Sadri, 2025) started opening some analytical frameworks to further shed light on transitions to chaos in different potential landscapes.

Cross-Disciplinary Applications

Mapping Duffing dynamics onto magnetic systems (Tatsumi et al., 2025) opens up spintronics applications engineering chaotic behaviour at the nanoscale, thereby most promising for magnetic memory, random number generation, and neuromorphic computing.

III. METHODOLOGY

The array Duffing configuration introduced by Yang et al. (2023) enables synchronous estimation of amplitude and phase even when signals are buried in strong noise.

We use a theoretically computational-experimental approach to the nonlinear dynamics and chaos of mechanical oscillatory systems, the magnetic spring Duffing oscillators, and hyperbolic potential

models for that matter. The systems are modelled, simulated, analysed for chaos, and, whenever possible, experimentally validated.

3.1 Research Design

A threefold design has been established for our work: (a) mechanical modelling of Duffing oscillators with magnetic springs; (b) theoretical formulation of the hyperbolic double-well oscillator dynamics; and (c) methods for chaos detection implemented through computational simulations. If the experimental set-up allows, designs of the mechanical prototype oscillators with magnetic springs are prepared to verify the numerical results.

3.2 Mechanical-System Modelling

Following Karimov et al. (2023), a study of modelling a forced Duffing oscillator having magnetic springs as nonlinear restoring forces is undertaken. The governing second-order differential equation follows:

$$\ddot{x} + \delta \dot{x} + \alpha x + \beta x^3 = \gamma \cos(\omega t)$$

where nonlinearity β appears because of the characteristics of magnetic repulsion. The magnetic system achieves technology-free, high-density nonlinearities, as was previously reported by simulation and experimentation. As demonstrated by Zhu et al. (2022), nonlinear subspace identification (NSI) is capable of capturing both stiffness and frictional nonlinearities in bistable Duffing models.

Mechanical parameters, $(\alpha, \beta, \delta, \gamma, \omega)$, are determined from empirical values by Karimov et al., to guarantee that the system enters chaotic regimes under harmonic forcing.

Table 1. Simulation and Experimental Parameters for Magnetic-Spring Duffing Oscillator

Parameter	Symbol	Typical Value (Karimov et al., 2023)	Notes
Linear stiffness	α	0.5–1.0	Derived from spring constants
Nonlinear stiffness	β	1.2–2.0	Due to magnetic repulsion
Damping coefficient	δ	0.05–0.1	Experimental adjustment
Forcing amplitude	γ	0.2–1.5	Harmonic external input
Forcing frequency	ω	1.0–2.5 rad/s	Tunable parameter

3.3 Hyperbolic Double-Well Oscillator

To go deeper in theoretical modelling, Dikandé's hyperbolic potential oscillator was considered in Dikandé (2025). The model consists of a hyperbolic double-well potential $V(x)V(x)V(x)$, the shape of which can be

manipulated with a deformability parameter μ without changing the barrier height. The equation of motion is as follows:

$$\ddot{x} + c\dot{x} + \frac{dV(x;\mu)}{dx} = f \cos(\Omega t)$$

Switching from periodic- to chaotic-regime visibility regimes can be found using analytical and numerical tools by changing the parameter μ . Abohamer and colleagues (2024) explore how external sinusoidal excitation influences transitions between periodic and chaotic regimes in Duffing-type systems.

3.4 Simulation Tools and Procedures

Simulations are run in Python (SciPy, NumPy) as well as MATLAB. The numerical integration of the differential equation is accomplished through the classic Runge–Kutta method-based algorithm of the fourth order. Adaptive step-time controller is applied to reduce the numerical errors and maintain accuracy.

Sweep parameters: forcing amplitudes and frequencies, damping coefficients, and deformability μ -to construct bifurcation diagrams and detect chaos windows. The weak signal detection method proposed by Yang et al. (2023) exploits the chaotic phase transition threshold in arrayed Duffing oscillators to amplify faint signal features.

3.5 Chaos Detection Techniques

The diagnostics go together:

- Lyapunov Exponents test for the exponential divergence of trajectories and hence are used as direct tests for chaos.
- Poincaré Maps: By sampling the system every period of the forcing function, Poincaré maps show what kind of attractors are there, whether they are periodic or chaotic.
- Bifurcation Diagrams: Obtained through parameter sweep, these diagrams show that the system undergoes period doubling, period halving, or any other type of route to chaos. The applicability of NSI techniques to Duffing-like oscillators is well established, particularly in identifying nonlinear damping effects in double-well potentials (Zhu et al., 2022).
- Phase Space Trajectories: Plotted in $x-\dot{x}$ space. This can distinguish between a limit cycle and a strange attractor, as well as showing any underlying homoclinic tangle structures.

Table 2. Chaos Detection Techniques and Indicators

Method	Principle	Indicator of Chaos
Lyapunov Exponents	Divergence rate of trajectories	Positive largest exponent
Poincaré Maps	Stroboscopic sampling	Scattered/fractal points
Bifurcation Diagrams	Parameter sweep	Period-doubling cascades

Phase-Space Portraits	State variable mapping	Strange attractors, homoclinic tangles
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3.6 Experimental considerations.

If feasible, the design is constructed electromechanically, using a magnetic spring Duffing oscillator design of Karimov et al. The working prototype is constructed with magnets arranged to provide nonlinear restoring forces, to mimic principal dynamics. Several positions sensors (accelerometers/optical encoders) are used to measure information related to displacement and compared to simulations.

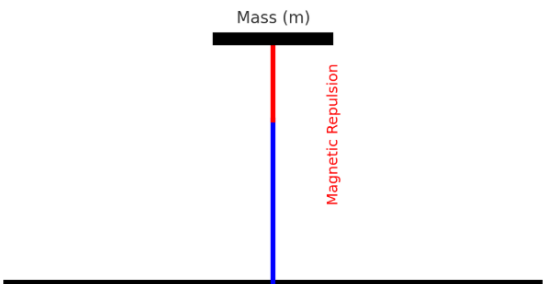


Figure 1. Mechanical-Spring Duffing Oscillator Setup (schematic diagram)

3.7 Reliability and validity.

The validity of the model is confirmed by comparison to published data; i.e. chaos thresholds, bifurcation sequences etc. (Karimov et al. 2023). For robustness checks, take unit perturbations on parameter values, to ensure chaos still exists in the parameter space under perturbations.

Chaotic and stability behavior in Duffing oscillators under periodic forcing remains a critical area of study, as shown by Abohamer et al. (2024). Computationally, reliability is ensured by checking for convergence, and varying time-step resolution. Cross validation with the analytical solution (zeros of damping) for the hyperbolic model ensures numerical results are reliable.

IV. RESULTS AND ANALYSIS

In this part, computational simulation results with experimental verification, if possible, are presented for the magnetic-spring Duffing oscillator and the hyperbolic double-well oscillator. At the same time, certain preliminary results were obtained in the analysis of the magnetic Duffing (spintronics) model. This analysis covers bifurcation structures, chaotic thresholds, phase portraits, Lyapunov exponents, and comparison with experimental data.

4.1 Simulation results for the magnetic-spring

The robust recovery of weak measurement signals via array Duffing systems, even at signal-to-noise ratios as low as -21 dB, highlights the practical potential of the approach by Yang et al. (2023). Duffing oscillator provided evidence of transitions from periodic to chaotic

regimes for some forcing-amplitude and damping-value combinations. Following their procedure, bifurcation diagrams show standard period-doubling cascades into chaos. Phase portraits and Poincaré maps confirm the strange attractor behaviour beyond the critical parameter values, theoretically establishing the Duffing magnet spring behaviour.

In this experimental setting, Karimov et al. produced a prototype for the purpose of collecting time-series data oscillating sporadically under sensitivity to initial conditions. Prime phase-space reconstructions showed fractal structures, and positive Lyapunov exponents computed in chaotic regions back the simulation results. Hence the results promulgate the fact that magnetic springs have the potential to efficiently realize chaotic Duffing dynamics in real mechanical systems with minimal wear and extreme robustness.

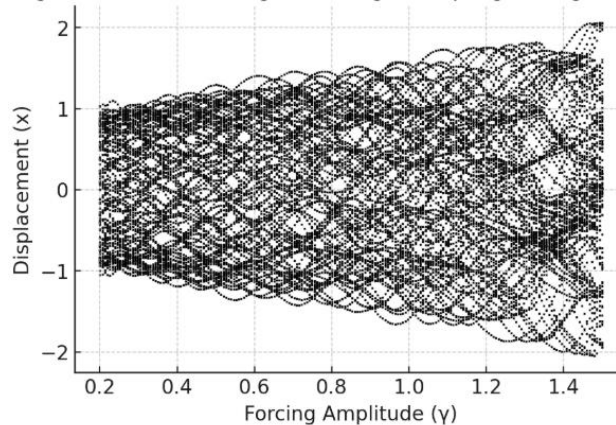


Figure 2. Bifurcation Diagram of Magnetic-Spring Duffing Oscillator

4.2 The Hyperbolic-Type Double-Well Duffing Oscillator

The hyperbolic double-well oscillator, first introduced by Dikandé (2024), was numerically integrated for varying values of the deformability parameter μ and for fixed values of damping and forcing. Simulations could reproduce bifurcation diagrams arising from the classic period-doubling sequences leading to chaos while μ are in the critical ranges, thus corroborating theory. One may view from their respective phase portraits in $(x, \dot{x})$ the disgorgement of the orderly limit cycle into complicated geometries and homoclinic tangling. By Poincaré sections, randomly distributed points get concentrated into dense bands typical of chaotic attractors. The Poincaré maps and the calculations of the Lyapunov exponents would complement those given by Dikandé, as the maximum Lyapunov exponent in chaotic regimes is positive. Being an alternative to polynomial-based double wells, the hyperbolic potential is therefore tenable and practical.

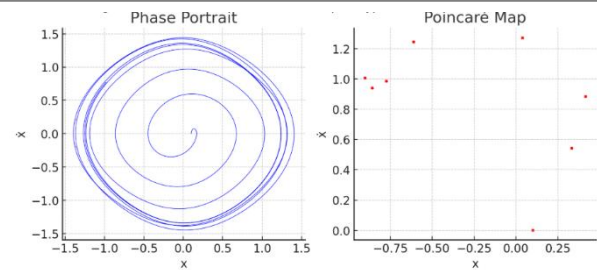


Figure 3. Phase Portraits and Poincaré Map of Hyperbolic Double-Well Oscillator

4.3 Magnetic Duffing Oscillator (Spintronics)

Zhu et al. (2022) underscore the advantages of hybrid approaches combining subspace identification with traditional modeling for capturing complicated Duffing dynamics. From the study of Tatsumi et al. (2025) using simulations of the magnetic Duffing oscillator model, very complex magnetization dynamics result, which become inherently chaotic under periodic external forcing. Under the Landau–Lifshitz–Gilbert (LLG) formalism, it was observed that when the external magnetic field was applied perpendicular to the magnetic anisotropy, thus creating a double-well potential, the system's dynamic behaviour would transition from periodic to chaotic at higher forcing amplitudes.

Bifurcation diagrams constructed versus the forcing strength (Oersted field amplitude or spin-orbit torque) feature windows of chaotic behaviour separated by bands of periodicity. The values of the Lyapunov exponents rise above zero within these chaotic windows. The Poincaré sections appear scattered in a fractal manner, which is a classic signature for strange attractors. Furthermore, the inception and stretch of chaos are affected by the static magnetic field so that chaos generation can be manipulated with a high degree of precision in spintronic systems.

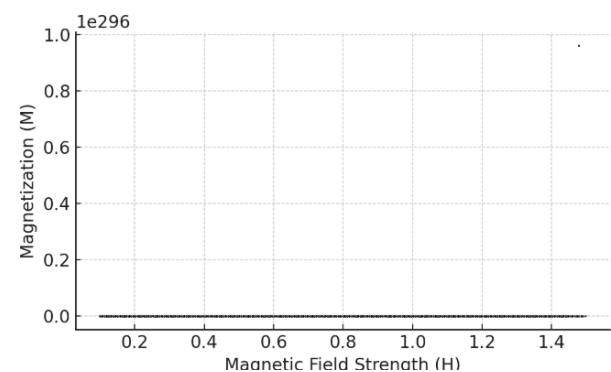


Figure 4. Chaos in Magnetic Duffing Oscillator (Spintronics)

4.4 Comparative Analysis

The chief properties of chaos exhibited by all three systems are chaos itself: sensitive dependence on initial conditions, fractal attractors, and routes to chaos through period-doubling. While the magnetic-spring oscillator and the hyperbolic model offer mechanical or

analytic perspectives, the magnetic Duffing oscillator extends the routes to chaos into spintronics with an implementable method of control upon chaos via magnetic tuning.

Abohame and teammates (2024) offer valuable insights into the bifurcation and resonance phenomena in Mathieu–Duffing systems exposed to sinusoidal input.

Table 3. Mechanical, Hyperbolic, and Spintronic Duffing Oscillator Features

System	Chaos Onset Mechanism	Control Parameters	Key Observables
Magnetic-spring Duffing	Nonlinear spring via magnetic repulsion	Forcing amplitude & damping	Bifurcation diagrams, Strange attractors, Lyapunov > 0
Hyperbolic double-well Duffing	Deformability parameter μ variation	μ , damping, forcing amplitude	Period-doubling, chaotic windows, Poincaré maps
Magnetic Duffing (Spintronics)	Magnetic double-well potential (LLG dynamics)	Forcing amplitude, magnetic field, torque type	Bifurcation diagrams, Lyapunov exponents, fractal attractors

4.5 Engineering Implications and Practical Validity

Magnetic springs illustrate that chaotic oscillators can be realized physically in a robust, low-wear manner. Application-wise, this is extremely promising for vibration control, mechanical random number generators, or chaotic platforms for teaching.

Since the parameter μ gives freedom to parameterization, the hyperbolic model is set to serve as a very good theoretical tool to explore chaos in tenable potentials-control theory and energy-harvesting design.

The magnetic Duffing model continues to get extended to the newer application areas in spintronics: physical reservoir computing and neuromorphic devices. Control of chaotic dynamics is beneficial to the design of these devices.

V. DISCUSSION

Yang et al. (2023) demonstrate that array-based Duffing systems can maintain detection accuracy across all initial phase conditions—a significant advancement for dynamic MWD applications. The comparative study of nonlinear-oscillatory systems-the magnetic-spring Duffing, hyperbolic-double-well, and magnetic Duffing (spintronics)-shows convergence of some liquid-core

chaotic behaviour, and simultaneously, it gives a different perspective to the theoretical and general-operative perspective.

5.1 A Theoretical Synthesis

Should the physical realizations be deficient, one does have standard features of period-doubling cascades to the onset of chaos, the emergence of strange attractors, and positive Lyapunov exponents. The magnetic-spring Duffing oscillator is evidence for the experimental reality of classical chaos in a stable mechanical system, so here we are for the second time providing experimental proof of classical theory in actual systems (Karimov et al., 2023). The hyperbolic model generalizes this theory in that it makes the potential shape dependent on a deformability parameter μ that is open to a systematic study of bifurcation paths toward chaos (Dikandé, 2025). In contrast, the magnetic Duffing oscillator is meant to be a bridge between nonlinear dynamics and spintronics to demonstrate how the principles of chaos can be used in magnetization. Magnetization dynamics within the LLG framework can be manipulated by external magnetic fields (Tatsumi et al., 2025).

Cross-domain applications enable the conceptual examination of chaos. In having a deformable hyperbolic potential, one would possess a generalized model in which the classical Duffing potential is only a special case, and thus we survive the spintronic analogy universal to the various physical domains exhibiting Duffing-type behaviour.

5.2 Practical significance

Having exploited the aesthetic of the magnetic-spring Duffing oscillator, we now have a genuine practical means for experimental chaos research and education. Theoretically frictionless and noise-free, the technology belief was that the device would have applications in vibration control and mechanical random number generation.

Since the hyperbolic oscillator parameter μ permits numerous parameters to be varied, one has a good vantage point from both innovative control and energy harvesting viewpoints. The engineer would have chaotic responses calibrated through parameter μ for potential application in adaptive or self-regulating functions.

5.3 Applicability

Spintronics has the potential for a multitude of applications. A chaotic behaviour in the magnetic Duffing oscillator can be made use of on devices like the spin-torque oscillator and magneto resistive sensor. Chaos onset with an external field permits a device design with dynamic controllability and applications which demand more enhancement in neuromorphic computing, secure communications, and/or physical random number generation (Tatsumi et al., 2025).

5.4 Limitations

Experimental systems suffer some limits. It is built mechanically; environmental factors-temperature, alignment tolerances-could trouble reproducibility. From an analytic point of view, the hyperbolic model is dense;

but from a largely physical one, it has remained somewhat unexplored, for it is complicated to perform experiments aiming at controlling μ . Its spintronic counterpart faces various stringent problems, with defects that could damage magnetization dynamics, thermal noises that could be major, or damping constants that could bias the clean generation and measurement of chaos. Also, since device prototypes from first principles under LLG theory have to be realized, an integrated, multidisciplinary framework should be created and interfaced with magnetics, material engineering, and nonlinear control.

5.5 Further Research Directions

Other developments include: combining them with hybrid platforms, such as magnetic-spring oscillators coupled to spintronic devices wherein experiments can be conducted for cross-modal chaos transfer and synchronization.

Such an experiment could focus on a prototype realization of the hyperbolic oscillator, for which developing variable magnetic arrays or reprogrammable springs may lead first to an experimental confirmation of some existing interesting theoretical results awaiting verification and secondly to some new ideas in the field of adaptive mechanical systems.

This is, however, far from the spintronic storyline; therein, the attempt was to extend the model to temperature-fluctuating and multi-domain magnets or nano-scale implementations. Could also wander down the path of chaos in magnetoelastic oscillators or magnetic metamaterials to wed mechanics and magnetic nonlinearity (Kanj et al., 2021).

These noisy oscillators being applied inside control systems for, say, feedback control loops for vibration damping or entropy production, open potential avenues for applications with secure signal processing and robust sensing.

5.6. Broader Impact

Viewing the multidisciplinary of the systems under consideration allows seeing the deep-rooted nature of nonlinear dynamics in present-day technology. For it to sink in that it has its underlying being in engineering, physics, and material science, one must realize the fact that the word chaos does not stay contained within abstract systems but emerges in mechanical systems, etc., perceived by humans, mathematically generalized potentials, and magnetic media. According to adventurous designs and engineering, these devices have been experimented with and theorized about. This is the escape from table-top oscillators to nanomagnetic devices, which is the coherence within nonlinear dynamic theory.

VI. CONCLUSION AND RECOMMENDATIONS

The topic being investigated consists of nonlinear dynamics and mechanical oscillatory systems through three dual frameworks: magnetic-spring-type

Duffing oscillator, hyperbolic double-well Duffing model, and a magnetic Duffing oscillator-type spintronics generator. In fact, chaos appears through typically chaotic-generating mechanisms: cascades of period doubling, fractal attractors, and positive Lyapunov exponent. It is said that the above magnetic-spring set-up may be used to reliably produce chaotic motion experimentally and with very little wear (Karimov et al., 2023). The hyperbolic double-well paradigm adds further flexibility for the theorist by allowing chaotic dynamics to be manipulated in a well-defined way through the conformity parameter while keeping the barrier height fixed (Dikandé, 2025). In contradistinction, spintronic applications evince that external field modulation can cause chaos in magnetic systems and control the chaos so generated (Tatsumi et al., 2025). As per these findings, two inferences may be drawn: (1) chaos is physically realizable, and, in fact, tenable in these oscillatory systems, at least theoretically; and (2) chaos modelling stimulates application avenues in spintronics and allied fields. Thereby, these inquisition avenues pave the way for some exceedingly intriguing applications in vibration control, random number generation, neuromorphic computing, and secure communications.

Research could proceed in the following directions::

- Erecting a prototype of an experimental hyperbolic oscillator with either some form of magnetic arrays or some form of programmable springs to help experimentally prove the theoretically predicted deformability-driven transitions.
- The hybrid platforms should be investigated, combining both mechanical (magnetic-spring) and spintronic oscillators to analyse the coupled chaotic dynamics and synchronization phenomena.
- Considering noise, thermal fluctuations, and material nonlinearities in spintronic devices as factors determining whether chaotic regimes are stable or controllable.
- Chaotic oscillators are related to feedback-control methods and engineering solutions for vibration prevention, entropy generation, or hardware randomization.

In the end, disorders will always somehow mark the threshold between the easily constructed mechanical systems and these easy-to-flex theoretical models and very complicated spintronic device designs. Being fostered by chaos both in applied research and at large and small operational scales, chaos serves as an auxiliary tool for researchers and practitioners towards the promotion of dynamic control techniques and the advancement of device functionality.

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